I'm not a robot

Did this video help you?Many real-life situations can be modelled using sequences and series, including but not limited to: patterns made when tiling floors; seating people around a table; the rate of change of a population; the spread of a virus and many more.What do I need to know about applications of arithmetic sequences and series?If a quantity is changing repeatedly by having a fixed amount added to or subtracted from it then the use of arithmetic sequences and arithmetic series is appropriate to model the situationIf a sequence seems to fit the pattern of an arithmetic sequence it can be said to be modelled by an arithmetic sequenceThe scenario can be modelled using the given information and the formulae from the formula bookletA common application of arithmetic sequences and series is simple interestSimple interest is when an initial investment is made and then a percentage of the initial investment is added to this amount on a regular basis (usually per year)Arithmetic sequences can be used to make estimations about how something will change in the futureExam questions won't always tell you to use sequences and series methods, practice spotting them by looking for clues in the questionIf a given amount is repeated periodically then it is likely the question is on arithmetic sequences or series Jasper is saving for a new car. He puts USD $100 into his savings account and then each month he puts in USD $10 more than the month before. Jasper needs USD $1200 for the car. Assuming no interest is added, find, i) the amount Jasper has saved after four months,ii) the month in which Jasper reaches his goal of USD $1200.Did this video help you?If a quantity is changing repeatedly by a fixed percentage, or by being multiplied repeatedly by a fixed amount, then the use of geometric sequences and geometric series is appropriate to model the situationIf a sequence seems to fit the pattern of a geometric sequence it can be said to be modelled by a geometric sequenceThe scenario can be modelled using the given information and the formulae from the formula bookletA common application of geometric sequences and series is compound interestCompound interest is when an initial investment is made and then interest is paid on the initial amount and on the interest already earned on a regular basis (usually every year)Geometric sequences can be used to make estimations about how something will change in the futureThe questions won’t always tell you to use sequences and series methods, so be prepared to spot ‘hidden’ sequences and series questionsLook out for questions on savings accounts, salaries, sales commissions, profits, population growth and decay, spread of bacteria etcExam questions won't always tell you to use sequences and series methods, practice spotting them by looking for clues in the questionIf a given amount is changing by a percentage or multiple then it is likely the question is on geometric sequences or series A new virus is circulating on a remote island. On day one there were 10 people infected, with the number of new infections increasing at a rate of 40% per day. a) Find the expected number of people newly infected on the 7th day.b) Find the expected number of infected people after one week (7 days), assuming no one has recovered yet.Did this page help you? Calculator-online.net is your ultimate destination for a wide range of free online calculators. These tools include AI tools, health, finance, statistics, maths, physics, and chemistry. Everyone deserves instant and free access to reliable calculations. Our mission is to provide accurate and up-to-date results to tackle challenges with precision. Why does Calculator Online strive to be a one-stop destination? The team of online calculators continues to update, and new tools are being added regularly. Whether you are tackling basic maths problems or delving into more complex equations, this platform is designed to streamline your mathematical endeavours. Every tool developed by the Calculator Online is designed for accurate results, showing our strong commitment to your satisfaction. If you feel any inconvenience regarding these online calculators, feel free to Contact Us. Increase your mathematical productivity with the Calculator Online, your best companion in mathematics. by:1 (Source: Crossref) Abstract: Power series are useful tools that can be used to expand other functions, solve equations, provide for assessment of intervals of convergence, used as trial functions, and are applied in all areas of engineering. Included in this discussion are Taylor's Series, which are extremely important in numerical approximations. We present here a brief discussion of Power Series with several applications. An important application of power series in the field of engineering is spectrum analysis. In radio, audio, and light applications, it is very useful to be able to receive a wide range of frequencies and be able to pinpoint which frequencies are the loudest/brightest. By using a form of power series called the Fourier Series/Fourier Transform, we are able to do this mathematically. This algorithm is used in everything from instrument tuners to deep-space telescopes, and learning power series is very important to understanding it. Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Learning Outcomes Combine power series by addition or subtraction Create a new power series by multiplication by a power of the variable or a constant, or by substitution Multiply two power series together If we have two power series with the same interval of convergence, we can add or subtract the two series to create a new power series, also with the same interval of convergence. Similarly, we can multiply a power series by a power of x or evaluate a power series at [latex]{x}^{m}[/latex] for a positive integer m to create a new power series. Being able to do this allows us to find power series representations for certain functions by using power series representations of other functions. For example, since we know the power series representation for [latex]f\left(x\right)=\frac{1}{1-x}[/latex], we can find power series representations for related functions, such as [latex]y=\frac{3x}{1-{x}^{2}}\text{ and }y=\frac{1}{\left(x - 1\right)\left(x - 3\right)}[/latex]. In Combining Power Series we state results regarding addition or subtraction of power series, composition of a power series, and multiplication of a power series by a power of the variable. For simplicity, we state the theorem for power series centered at [latex]x=0[/latex]. Similar results hold for power series centered at [latex]x=a[/latex]. Suppose that the two power series [latex]\displaystyle\sum _{n=0}^{\infty }{c}_{n}{x}^{n}[/latex] and [latex]\displaystyle\sum _{n=0}^{\infty }{d}_{n}{x}^{n}[/latex] converge to the functions f and g, respectively, on a common interval I. The power series [latex]\displaystyle\sum _{n=0}^{\infty }\left({c}_{n}{x}^{n}\pm {d}_{n}{x}^{n}\right)[/latex] converges to [latex]f\pm g[/latex] on I. For any integer [latex]m\ge 0[/latex] and any real number b, the power series [latex]\displaystyle\sum _{n=0}^{\infty }b{x}^{m}{c}_{n}{x}^{n}[/latex] converges to [latex]b{x}^{m}f\left(x\right)[/latex] on I. For any integer [latex]m\ge 0[/latex] and any real number b, the series [latex]\displaystyle\sum _{n=0}^{\infty }{c}_{n}{\left(b{x}^{m}\right)}^{n}[/latex] converges to [latex]f\left(b{x}^{m}\right)[/latex] for all x such that [latex]b{x}^{m}[/latex] is in I. We prove i. in the case of the series [latex]\displaystyle\sum _{n=0}^{\infty }\left({c}_{n}{x}^{n}+{d}_{n}{x}^{n}\right)[/latex]. Suppose that [latex]\displaystyle\sum _{n=0}^{\infty }{c}_{n}{x}^{n}[/latex] and [latex]\displaystyle\sum _{n=0}^{\infty }{d}_{n}{x}^{n}[/latex] converge to the functions f and g, respectively, on the interval I. Let x be a point in I and let [latex]{S}_{N}\left(x\right)[/latex] and [latex]{T}_{N}\left(x\right)[/latex] denote the Nth partial sums of the series [latex]\displaystyle\sum _{n=0}^{\infty }{c}_{n}{x}^{n}[/latex] and [latex]\displaystyle\sum _{n=0}^{\infty }{d}_{n}{x}^{n}[/latex], respectively. Then the sequence [latex]\left\{{S}_{N}\left(x\right)\right\}[/latex] converges to [latex]f\left(x\right)[/latex] and the sequence [latex]\left\{{T}_{N}\left(x\right)\right\}[/latex] converges to [latex]g\left(x\right)[/latex]. Furthermore, the Nth partial sum of [latex]\displaystyle\sum _{n=0}^{\infty }\left({c}_{n}{x}^{n}+{d}_{n}{x}^{n}\right)[/latex] is [latex]\begin{array}{cc}\hfill {\displaystyle\sum _{n=0}^{N}} \left({c}_{n}{x}^{n}+{d}_{n}{x}^{n}\right)& ={\displaystyle\sum _{n=0}^{N}} {c}_{n}{x}^{n}+{\displaystyle\sum _{n=0}^{N}}{d}_{n}{x}^{n}\hfill \\ & ={S}_{N}\left(x\right)+{T}_{N}\left(x\right).\hfill \end{array}[/latex] Because [latex]\begin{array}{cc} \hfill {\underset{N\to\infty}\lim} \left({S}_{N}\left(x\right)+{T}_{N}\left(x\right)\right) & ={\underset{N\to\infty}\lim} {S}_{N}\left(x\right)+{\underset{N\to\infty}\lim} {T}_{N}\left(x\right)\hfill \\ & =f\left(x\right)+g\left(x\right),\hfill \end{array}[/latex] we conclude that the series [latex]\displaystyle\sum _{n=0}^{\infty }\left({c}_{n}{x}^{n}+{d}_{n}{x}^{n}\right)[/latex] converges to [latex]f\left(x\right)+g\left(x\right)[/latex]. [latex]\blacksquare[/latex] We examine products of power series in a later theorem. First, we show several applications of Combining Power Series and how to find the interval of convergence of a power series given the interval of convergence of a related power series. Suppose that [latex]\displaystyle\sum _{n=0}^{\infty }{a}_{n}{x}^{n}[/latex] is a power series whose interval of convergence is [latex]\left(-1,1\right)[/latex], and suppose that [latex]\displaystyle\sum _{n=0}^{\infty }{b}_{n}{x}^{n}[/latex] is a power series whose interval of convergence is [latex]\left(-2,2\right)[/latex]. Find the interval of convergence of the series [latex]\displaystyle\sum _{n=0}^{\infty }\left({a}_{n}{x}^{n}+{b}_{n}{x}^{n}\right)[/latex]. Find the interval of convergence of the series [latex]\displaystyle\sum _{n=0}^{\infty }{a}_{n}{3}^{n}{x}^{n}[/latex]. Watch the following video to see the worked solution to Example: Combining Power Series.In the next example, we show how to use Combining Power Series and the power series for a function f to construct power series for functions related to f. Specifically, we consider functions related to the function [latex]f\left(x\right)=\frac{1}{1-x}[/latex] and we use the fact that[latex]\frac{1}{1-x}=\displaystyle\sum _{n=0}^{\infty }{x}^{n}=1+x+{x}^{2}+{x}^{3}+\cdots[/latex]for [latex]|x| Use the power series representation for [latex]f\left(x\right)=\frac{1}{1-x}[/latex] combined with Combining Power Series to construct a power series for each of the following functions. Find the interval of convergence of the power series. [latex]f\left(x\right)=\frac{3x}{1+{x}^{2}}[/latex] [latex]f\left(x\right)=\frac{1}{\left(x - 1\right)\left(x - 3\right)}[/latex] Show Solution A power series will converge only for certain values of . For instance, converges for . In general, there is always an interval in which a power series converges, and the number is called the radius of convergence (while the interval itself is called the interval of convergence). The quantity is called the radius of convergence because, in the case of a power series with complex coefficients, the values of with form an open disk with radius . A power series always converges absolutely within its radius of convergence. This can be seen by fixing and supposing that there exists a subsequence such that is unbounded. Then the power series does not converge (in fact, the terms are unbounded) because it fails the limit test. Therefore, for with , the power series does not converge, where and denotes the supremum limit. Conversely, suppose that . Then for any radius with , the terms satisfy for large enough (depending on). It is sufficient to fix a value for in between and . Because , the power series is dominated by a convergent geometric series. Hence, the power series converges absolutely by the limit comparison test. Power series in mathematics is an infinite series of the form $\sum_{n=0}^{\infty}a_n (x - c)^n$. Where a_n is the nth term coefficient and c is a constant.$\sum_{n=0}^{\infty}a_n(x - c)^n = a_0 + a_1(x - c)^1 + a_2(x - c)^2 + a_3(x - c)^3 + ...$Any polynomial can be expressed as a power series around any c, but all but a small number of the coefficients will be zero since a power series by definition has infinitely many terms. When the absolute value of x is less than some positive number r, known as the radius of convergence, a given power series will usually converge, that is, the power series will reach a finite sum for all the given values of x within a certain interval around zero in particular. Outside of this interval, the series diverges, reaching an infinite number, while when $x = \pm r$, the series can converge or diverge. A variant of the ratio test for power series may also be used to evaluate the radius of convergence.Power Series ExamplesEvery polynomial can be expressed as a power series around c, but all but a small number of the coefficients will be zero because, by definition, a power series has infinite terms. For example the polynomial $f(x) = x^3 + 7x^2 + 4x + 3$ can be written in a power series around the $c = 0$ as follows:$f(x) = 3 + 4x + 7x^2 + 1x^3 + 0x^4 + 0x^5 + ...$The power series formula for a geometric series formula will be as follows: $\dfrac{1}{1 - x} = \sum_{n=0}^{\infty}x^n = 1 + x + x^2 + x^3 + x^4 + ... $Power series in mathematics for an exponential function is $e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} = 1 + \dfrac{x}{1} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + ...$Power series in mathematics for sine functions is$\sin(x) = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}= x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \dfrac{x^9}{9!} -.....$Power series in mathematics for cosine function is $\cos(x) = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{2n!} = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \dfrac{x^8}{8!} - .....$The power series in mathematics of a logarithmic function is $ln(1+x) = \sum_{n=0}^{\infty} \dfrac{(-1)^{n-1} x^n}{n} = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \dfrac{x^5}{5} - .....$The power series in mathematics of an inverse tangent function is $tan^{-1}(x) = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{2n+1} = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \dfrac{x^9}{9} - .....$Radius of ConvergenceFor certain values of the variable x, such as $x = c$, a power series is convergent. For other x values, the series can diverge. If c isn't the only point of convergence, there's always a number r with $0 < r \leq \infty$ such that the series converges when $|x – c| < r$ and diverges when $|x – c| > r$. The radius of convergence of the power series is denoted by the number r. The radius of convergence r for the power series is given as follows,$r = \lim_{n \to \infty} inf |an|^{-\frac{1}{n}}$ Or $r^{-1} = \lim_{n \to \infty} sup |a_n|^{\frac{1}{n}} $Here $\lim\,inf$ and $\lim\,sup$ are limit inferior and limit superior which are limiting bounds on the sequence.Operations on Power SeriesWe can perform basic operations on power series along with the complex calculus operations too. Here let us have a look at a few of the important operations performed on the power series.Addition and Subtraction of the Power SeriesWhen two functions f and g are decomposed into power series around the same centre c, termwise addition and subtraction can be used to obtain the power series of the sum or difference of the functions.If $f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$ and $g(x) = \sum_{n=0}^{\infty} b_n(x-c)^n$ are two power series then the addition and subtraction are as follows:$f(x) \pm g(x) = \sum_{n=0}^{\infty}(a_n \pm b_n)(x-c)^n$Multiplication and Division of the Power SeriesIf $f(x) = \sum_{n=0}^{\infty}a_n(x - c)^n$ and $g(x) = \sum_{n=0}^{\infty}b_n(x - c)^n$ are two power series then the multiplication of the two power series is as follows:$f(x) g(x) = \left(\sum_{n=0}^{\infty}a_n(x-c)^n \right) \left(\sum_{n=0}^{\infty} b_n(x-c)^n \right) \\ f(x) g(x) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_i b_j (x-c)^{i+j} \\ f(x) g(x) = \sum_{n=0}^{\infty} \left(\sum_{i=0}^{\infty} a_i b_{n-i} \right) (x-c)^n$The sequence $\sum_{i=0}^{\infty} a_ib_{n-i}$ is known as the convolution of the sequences a_n and b_n.If $f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$ and $g(x) = \sum_{n=0}^{\infty} b_n(x-c)^n$ are two power series then the division of the two power series is as follows:$\dfrac{f(x)}{g(x)} = \dfrac{ \sum_{n=0}^{\infty} a_n(x-c)^n}{\sum_{n=0}^{\infty} b_n(x-c)^n} = \sum_{n=0}^{\infty} d_n(x-c)^n$Differentiation and Integration of the Power SeriesWhen a function $f(x)$ is expressed as a power series $\sum_{n=0}^{\infty} a_n(x-c)^n$, it can be differentiated on the interior of the convergence domain. It's simple to differentiate and integrate by treating each term separately.Differentiation of the power series $f(x)\sum_{n=0}^{\infty} a_n(x-c)^n$ is as follows:$f^1(x)\sum_{n=1}^{\infty} a_n n(x-c)^n = \sum_{n=0}^{\infty} a_{n+1}(n+1)(x-c)^n$Integration of the power series $f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$ is as follows:$\int{f(x)\,dx} = \sum_{n=0}^{\infty} \dfrac{a_n(x-c)^{n+1}}{n+1} = \sum_{n=1}^{\infty} \dfrac{a_{n-1}(x-c)^n}{n} +k$Applications of Power SeriesPower series can be found as the Taylor series of infinitely differentiable functions in mathematical analysis. Every power series, according to Borel's theorem, is the Taylor series of some smooth function.Power series appear as generating functions in combinatorics and as the Z-transform in electronic engineering, in addition to their role in mathematical analysis.An example of a power series is also the well-known decimal notation for real numbers.The definition of p-adic numbers is closely related to that of a power series in number theory.ConclusionPower series are useful tools that can be used to extend other functions, solve equations, test convergence intervals, and serve as trial functions in a variety of engineering applications. Taylor's Series, which are extremely significant in numerical approximations, use power series. In the real world what are power or taylor series used for? Historically were they used for anything? Especially were they used for anything interesting? CRGreathouse In the real world what are power or taylor series used for? Calculating things that are too complicated to calculate in other ways, or working mathematically with objects that are too complicated to analyze in other ways. In the real world what are power or taylor series used for? Historically were they used for anything? Especially were they used for anything interesting? They have quite a variety of uses. Let's look at a few examples: 1) The transcendental functions (ie sin, cos, tan, log, exp etc) We know from taylor series that we can represent a function by the rgelationship to its derivatives and function value at a point. Now we don't know how to calculate sin(x) or cos(x) but we know the derivatives of these functions and their values at x = 0. Using a special case of taylor series (called a mclaurin series) we can find an expression for sin(x) when x 0 using knowledge about the differential at various degrees. So all of the transcendental functions can be calculated to find the value to any desired approximation. Also you should note that any function that has infinite terms has the potential to have infinite stationary points (turning points or points of inflection), so anything that is periodic over an infinite domain is basically a series. This brings me to part 2: 2) Fourier series: Fourier series builds on the idea that we can take things from the time domain and put them into the frequency domain. A lot of functions that a periodic over the reals have surprising simple series representations. Examples of this include the sawtooth function, the "clock" function, the signum function and so on. All of the above functions can be represented by infinite series and we can get as good approximations as we want to these with series expressions. 3) Systems in math and nature: The fact is that a lot of different systems do not have a closed form answer: they can be written in terms of infinite series. One surprising kind of math that uses an infinite series is called the Riemann Zeta Function. It has connections everywhere including number theory and even physics. There is a one million dollar reward to prove that the non trivial zeroes have real part = 1/2. If you look at many areas of science (including physics) you will see many examples of systems that have these so called series expansions. I hope that gives some insight to what is out there with series nothing can done without them, not even calculus. This power series calculator allows you to expand a function into a power series for a given variable and also shows you the step-by-step calculations involved. With it, you can: Perform the power series expansion for a function Specify the point (center) around which you want to expand Setting the order (n) of the expansion This calculator is very helpful for analyzing and approximating the function values. Limitation: The calculator can handle a range of mathematical functions, but this might not work for functions with discontinuities or infinite complexity. What is Power Series? "A power series is an infinite series where every single term is formed by the multiplication of a constant coefficient (cn) and a variable (x) raised to an increasing non-negative integer power (n), often centered around a value 'a'.” Power Series Formula: ∑n=0∞ cn(x-a)n = c0 + c1(x-a) + c2(x-a)2 + c3(x-a)3 + ...; Where: “x” is the variable “n” is a non-negative integer that shows the power “cn” represents constant coefficients “a” is a constant that indicates the center of power series When the value “a” is zero, then the series becomes simpler as: ∑n=0∞ cnxn = c0 + c1x + c2x2 + c3x3 + ...; This series proceeds to represent a function within the interval of convergence, and it diverges outside the interval. They are used: To define new functions and provide alternative representations of common functions The power series is valuable for approximating functions Solve differential equations Evaluate integrals Power Series Convergence: A power series centered at a point converges for a value of x within a certain interval. Within this interval of convergence, the absolute value of the terms typically decreases as the power increases. Hence, their sum approaches a finite value. This power series convergence can be found by using the ratio test. ∑(xn) = 1 + x + x2 + x3 + ... ∑n=0∞ x^n The power series converges when the absolute value |x| < 1. At this point, its value becomes 1/(1-x). We can express the function as power series manually, as we have done below. To quickly obtain its power series representation, try our power series calculator, which takes a function as input and outputs its equivalent power series. f(x) = 1/(1 - x) = Σ(n=0 to ∞) xn = 1 + x + x2 + x3 + ..., |x| < 1 This equation shows that the series converges at a certain value, and we can get another function by replacing x with -x f(x) = 1 / (1 + x) = ∑n=0∞ $(−x)^n$ = 1 − x + x^2 − x^3 + ... |−x|